DS APERIODIC: LYON, 2015

Akin *Subshifts via the Space of Labels and Weakly Almost Periodic Systems*

Abstract: The Space of Labels provides a new method of constructing examples of subshifts. In particular, we construct countable, topologically transitive WAP subshifts with some interesting properties. This is joint work with Eli Glasner.

Aujogue *Ellis semigroup for cut and project tilings*

Barge *The Pisot Conjecture for β-substitutions*

Abstract: We will sketch a proof that the tiling dynamical system $(\Omega_{\Psi_\beta}, \mathbb{R})$ associated with the β-substitution Ψ_β has pure discrete spectrum for every Pisot number β.

Boronski and **Oprocha** *Pseudocircle and attractors in the plane*

Abstract: In 1990 Barge and Martin presented a method of construction of global attractors of planar homeomorphisms in terms of inverse limits. This technique can also be extended to obtain attractors arising as inverse limits of graph maps, and even further to parametric families and is now being systematically studied under the name of BBMs. That way one can obtain attractors with very strange topological structure, such as pseudoarc or pseudocircle. Another common technique are Anosov-Katok type constructions, applied by Handel to obtain the pseudo-circle as a minimal set of a planar diffeomorphism, and later by Chritat to obtain it as the boundary of a Siegel disk.

In the first part of the talk we will survey results on construction of attractors in the plane, various types of continua, and general results on dynamics of these objects. We will also recall basic facts from rotation theory and the so-called Franks-Misiurewicz Conjecture. In the second part we will mainly focus on dynamics on the pseudo-circle when it arises as an attractor. Among other things, we will discuss some recent results on entropy and mixing in these systems, obtained recently by the authors, relating to the BBMs and Handel's construction.

Fokkink *The determinant of Hankel matrices of the period doubling sequence*

Abstract: The determinant of a Hankel matrix of a symbolic sequence measures its distance to a periodic sequence. It can be thought of as a symbolic equivalent of the irrationality measure and as such it has been used to study transcendence of automatic numbers. The specific values of the determinants are not easy to determine. For instance, the determinants of Hankel matrices of the Thue Morse sequence are not yet well understood. In this talk I will show that determinants for the period doubling sequence are products of Jacobsthal numbers. This is joint work with Cor Kraaikamp and Jeffrey Shallit.

Frank *Fractal dual substitution tilings*

Gaehler *Non-abelian invariants of tiling spaces*

Abstract: A tiling space X constructed as an inverse limit of approximants X_i and continous maps between them, has an associated inverse system consisting of the (pointed) fundamental groups $\pi_1(X_i)$ of the approximants and the induced homomorphisms between them. Clark and Hunton [1] have remarked that there is an invariant $L(X)$ associated with this inverse system, which is however difficult to compute (unless it is trivial). Sadun [2] has proposed an easier way to probe this inverse system, by looking at the corresponding direct limit of the dual objects $Hom(\pi_1(X_i), G)/G$, where G is any group. If G is abelian, this reduces to cohomology with coefficients in G, but the invariant proves much more powerful if a non-abelian group G is chosen. We illustrate on a number of examples, how these non-abelian invariants can distinguish many tiling spaces whose (even ordered) cohomology is the same.

[1] A. Clark and J. Hunton, New York J. Math. 18 (2012), 765-796.
[2] L. Sadun, Erg. Th. & Dyn. Sys. 27 (2007), 1991-1998.

Glasner *Enveloping semigroups and tame dynamical systems*

Abstract: According to a dynamical version of the Bourgain-Fremlin-Talagrand dichotomy, an enveloping semigroup of a dynamical system is either tame: has cardinality $\leqslant 2^{\aleph_0}$, or it is topologically wild and contains a copy of $\beta\mathbb{N}$, the Čech-Stone compactification of a discrete countable set. I will review the Ellis theory of enveloping semigroups and then focus on the theory of WAP, HAE and tame dynamical systems. Towards the end of the talk I'll describe the close connections of this dynamical theory with the geometry of Banach spaces. Mostly these are joint works with Michael Megrelishvily.

Julien *Gabor frames for cut-and-project sets*

Abstract: I will explain some of the results that Mike Kreisel obtained on Gabor frames for decorated lattices. The central object in his approach is a C*-algebra associated with the tiling groupoid, which a (literal) twist. I will then explain how to generalize his results to more general quasicrystals: any point-set can be deformed to a subset of a lattice, but the 2-cocycle still needs to be controlled. Some of these results can be related to a twisted version of the gap-labeling conjecture.

Lawson *Pseudogroups, Boolean inverse monoids and étale groupoids*

Abstract: In this talk, I shall explore the relationships between three kinds of structures: inverse monoids, groups and étale groupoids. Of these, the étale groupoids are probably the most familiar to those working in the theory of aperiodic tilings but under a generalization of classical Stone duality to a non-commutative setting there is a tight connection with (classes of) inverse monoids and, via their groups of units, with groups. I have four goals in this talk: to explain the inuitive background to these results, to outline the main theorems, to describe some important examples and applications (including recent contributions by Fred Wehrung) and finally to pose some

questions. This research was carried out in collaboration with Ganna Kudryavtseva (Ljubljana), Daniel Lenz (Jena), Pedro Resende (Lisbon), and Phil Scott (Ottawa). Its ultimate origins can be found in the work of Charles Ehresmann mediated through that of Renault, Kumjian, Kellendonk and Paterson (to name just a few).

Lenz *Self-similar groups and spectral theory of quasicrystals*

Abstract: We study spectral properties of the Laplacians on Schreier graphs arising from Grigorchuk's group acting on the boundary of the infinite binary tree. We establish a connection between the underlying dynamical system and a subshift associated to a non-primitive substitution and relate the Laplacians on the Schreier graphs to discrete Schroedinger operators with aperiodic order. We use this relation to prove that the spectrum of the anisotropic Laplacians is a Cantor set of Lebesgue measure zero. We also use it to show absence of eigenvalues both almost-surely and for certain specific graphs (joint work with Rostislav Grigorchuk and Tatiana Nagnibeda).

Minervino *Non-stationary Markov partitions for Pisot cocycles*

Abstract: A toral automorphism always admits a Markov partition. We generalize this result to the so-called non-stationary case, that is, when considering a bi-infinite sequence of toral automorphisms which are not necessarily the same. This corresponds to deal with a so-called symbolic renormalisation cocycle specified by a bi-infinite sequence of unimodular substitutions. Under certain assumptions of special hyperbolicity of the cocycle, namely the Pisot assumption, we can construct explicit Markov partitions by using the geometric theory of Rauzy fractals. We explore in particular the connections with multi-dimensional continued fraction algorithms, and we apply our theory to the case of the Brun substitutions.

Petite *Simplicity of the homeomorphisms groups of the hull of a repetitive Delone set*

Abstract: In a joint work with J. Aliste, we show that the identity component of the group of homeomorphisms that preserve each leaf (or $\mathbb{R}^d$–orbit) of the hull of a repetitive Delone set is simple. Moreover, in the one dimensional case, we show that this group is uniformly perfect. Indirectly we also get this group is a complete invariant of flow-equivalence. We obtain similar results for homeomorphisms preserving the Cantor transversal structure.

Richard *Dynamics on the graph of the torus parametrisation*

Abstract: Model sets are projections of certain lattice subsets. It was realised by Moody that dynamical properties of model sets are induced from the torus associated with the underlying lattice. We revisit this approach by studying dynamics on the graph of the map which associates lattice subsets to points of the torus and then transferring the results to their projections. This not only leads to transparent proofs of known results on model sets, but we also obtain new results on weak model sets. This is joint work with Gerhard Keller (Erlangen).

Rust *Non-minimal Substitution Tiling Spaces and Cohomology*

Abstract: Primitive substitutions have been well studied, and building inverse limit representations and calculating the cohomology of their associated tiling spaces is now well-understood. For simplicity, well only consider 1-dimensional substitutions. Well address the question of how necessary the assumption of primitivity is in this story and how, by dropping this assumption, we lose almost nothing in terms of the already-established topological theory, but enrich the number of examples that we can study. By considering those aperiodic substitutions whose tiling spaces are not minimal under the translation action, well describe a modified Anderson-Putnam method for building an inverse limit representation and for calculating a stratified version of cohomology. This stratified cohomology in some sense encodes the way in which the space is non-minimal by identifying and isolating the closed shift invariant subspaces of the tiling space. This invariant allows us to topologically distinguish between non-minimal tiling spaces whose ordinary cohomology groups are isomorphic. This is joint work with Greg Maloney.

Sadun *Homeomorphisms of tiling spaces*

Abstract: Let Ω_1 and Ω_2 be tiling spaces with finite local complexity, with Ω_1 uniquely ergodic. We show that every homeomorphism $h : \Omega_1 \to \Omega_2$ is homotopic to the composition of a shape change on Ω_1 followed by an MLD equivalence. To do this, we define a cohomological invariant that classifies homeomorphisms up to homotopy and MLD equivalence, and then show that all possible values of the invariant are achieved by shape changes. Similar results apply when Ω_1 or Ω_2 have infinite local complexity, only with "MLD equivalence" replaced by "topological conjugacy", and with the added assumption that the homeomorphism preserves translational orbits. Finally, these results also apply to maps between (FLC or ILC) tiling spaces that are not necessarily homeomorphisms, with "MLD equivalence" or "topological conjugacy" replaced by "local derivation" or "factor map". This is all joint work with Antoine Julien, building on one of Antoine's previous papers.

Weiss *Single-orbit spectral sets*

Abstract: We identify the closed support of the usual spectral measure for a bounded ergodic stationary process (over Γ) with the spectral set (in the sense introduced by Beurling) of a.e. sample considered as an element of $l^\infty(\Gamma)$. Here Γ can be any discrete abelian group like $\mathbb{Z}^d$. In fact the result is probably true for any locally compact abelian group.

Whittaker *Fractal spectral triples for substitution tilings*

Abstract: I will introduce a new class of noncommutative spectral triples on Kellendonk's $C*$–algebra associated with a nonperiodic substitution tiling. These spectral triples are built from fractal trees on tilings, which define a geodesic distance between tiles in self-similar tilings associated with each prototile. To elucidate our results I will construct a fractal tree on the Penrose tiling and explicitly show how it gives rise to a collection of spectral triples. This was joint work with Michael Mampusti, building on previous work with Sam Webster and Natalie Frank.